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Laminar film condensation on a horizontal wavy plate embedded in a porous medium

Tong Bou Chang *

Department of Mechanical Engineering, Southern Taiwan University of Technology, 1, Nai-Tai Street, YungKang City, Tainan County, Taiwan, ROC

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Abstract

The problem of two-dimensional, steady-state film condensation on an isothermal finite-size horizontal wavy plate embedded in a porous medium is studied for the case in which the plate faces upward into a region of dry saturated vapor. Due to capillary effects, a two-phase zone is formed between the liquid film on the cold plate and the vapor zone. Taking the sand–water–steam system for illustration purposes, the results indicate that the inclusion of capillary effects in the liquid film analysis has a significant effect on the computed results for the heat transfer coefficient. The results also reveal that the wave number and the wave amplitude of the wavy plate both have a significant effect on the mean Nusselt number, $\overline{Nu}$.

Keywords: Capillary effect; Wavy plate; Porous medium

1. Introduction

The problem of heat and mass transfer in a condensation layer within a porous medium has received extensive attention due to its wide range of practical applications in such diverse fields as evaporative condensers, heat pipes, geothermal energy utilization, thermally enhanced oil recovery, and so forth.

The heat transfer rates of laminar film condensation on vertical or nearly vertical surfaces were first predicted by Nusselt [1] in 1916. In Nusselt's analysis, the condensate film was assumed to be thin and convective and to have negligible inertial effects. Furthermore, the temperature profile within the condensate film was assumed to be linear. As summarized by Merte in [2], many researchers [3–8] have attempted to improve the accuracy of Nusselt's original analysis by removing its overly restrictive assumptions. The rate of heat transfer in a condensate film on a horizontal surface was first considered by Popov [9] in 1951. In a later study, Gerstmann and Griffith [10] employed both theoretical and experimental approaches to investigate condensation on the underside of a horizontal plate. The problem of condensation on the upper surface of a horizontal plate was first studied by Nimmo and Leppert [11,12]. However, in their analysis, the thickness of the condensate layer at the edge of the plate was either assigned an assumed value or was specified using a particular boundary condition. In more recent studies, Yang and Chen [13] and Chiou and Chang [14] applied the minimum mechanical energy principle [15] to establish the boundary conditions at the edge of the horizontal plate. The problem of downward condensate flow along a cool vertical or inclined surface embedded in a porous medium has been extensively studied [16–20]. Recently, Wang et al. [21–23] performed a series of theoretical investigations into steady filmwise condensation on a horizontal plate in a porous medium. Their results revealed that the rate of heat transfer in a condensation layer on a surface embedded in a porous medium can be enhanced by employing a wavy surface rather than a flat surface and applying a suction effect at the wall.

In general, the theoretical investigations described above neglected the effects of capillary forces in the porous medium. However, in practical problems involving condensation in porous media, the effective pore radii are very small. Consequently, capillary forces have a significant effect on the heat transfer performance and must be taken into consideration.

* Tel.: +886 6 2533131 x 3533; fax: +886 6 2425092.
E-mail address: tbchang@mail.stut.edu.tw.

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Nomenclature

a	wave amplitude defined in Eq. (1)
Bo_c	ratio of surface tension force to gravity force defined in Eq. (22)
Cp	specific heat at constant pressure
Da	Darcy number defined in Eq. (22)
F	dimensionless wave amplitude defined in Eq. (22)
g	acceleration of gravity
h	heat transfer coefficient
h_{fg}	heat of vaporization
Ja	Jakob number defined in Eq. (22)
k	thermal conductivity
K	permeability of porous medium
L	half-width of plate
$\dot{m}$	condensate mass flux
n	wave number
Nu	Nusselt number defined in Eq. (29)
P	pressure
Pr_e	effective Prandtl number defined in Eq. (22)
Ra_e	effective Rayleigh number defined in Eq. (22)
s	dimensionless saturation
T	temperature
ΔT	saturation temperature minus wall temperature
u, v	horizontal and vertical velocity components
w	surface function of wavy plate

Greek symbols

δ	condensate film thickness
δ_0	condensate film thickness at plate center
μ	liquid viscosity
ρ	liquid density
α	thermal diffusivity
σ	surface tension
ε	porosity

Superscripts

–	average quantity
$*$	dimensionless variable

Subscripts

2	properties in two-phase zone
o	quantity at plate center
c	capillary
min	minimum quantity or quantity at plate edge
sat	saturation property
w	quantity at wall
e	effective property

Using a sand–water–steam system, Udell [24,25] performed a series of theoretical and experimental investigations to examine the effect of capillary forces on heat transfer in porous media saturated with both vapor and liquid phases. Majumdar and Tien [26] studied the effect of capillary forces on film condensation along a vertical wall. Bridge et al. [27] investigated one-dimensional steady-state condensation heat and mass transfer in a two-phase zone by extending the system presented by Udell [24] to include an energy equation. More recently, Chang [28] investigated the effects of capillary forces on laminar film condensation on a horizontal disk embedded in a porous medium.

For condensation on a cold surface, increasing the contact area of the condensate provides an effective means of enhancing the heat transfer performance during condensation. Accordingly, various researchers have considered the use of finned surfaces to enhance the condensation heat transfer coefficient [29–31]. Yang et al. [32] and Wang et al. [22] have shown that a wavy surface results in an improved heat transfer performance compared to that provided by a flat surface since the former results in a larger contact area. It is therefore possible that wavy plates may replace the flat plates currently used in such diverse applications as waste disposal, chemical adsorption, heat pipe design, geothermal energy utilization, and distillation [33]. However, the literature contains very few reports addressing the problem of filmwise condensation on a horizontal wavy plate in a porous medium subject to capillary effects. Accordingly, the present study considers the case of saturated vapors condensing on a finite-size horizontal wavy plate embedded in a porous medium and maintained at a constant temperature. Specifically, the present analysis considers the influence of the Darcy number Da, the Jakob number Ja, the effective Rayleigh number Ra_e, and the effective Prandtl number Pr_e on the dimensionless mean Nusselt number. Additionally, taking the same sand–water–steam system as that considered by Udell [24], the effects on the heat transfer coefficient of capillary forces and the surface geometry of the horizontal wavy plate are systematically examined.

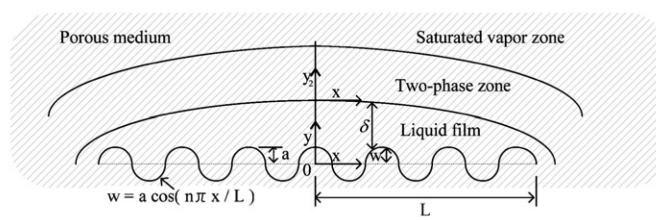

Fig. 1. Condensate film flow on finite-size horizontal wavy plate.

2. Analysis

Consider a horizontal wavy plate of finite size with a wall temperature T_w embedded in a porous medium filled with a dry vapor (see Fig. 1). As shown, the surface geometry function $w(x)$ of the wavy plate is given by

$$w(x) = a\cos(n\pi x/L) \tag{1}$$

where L is the half-width of the plate and a is the amplitude of the wavy surface.

If the clean wavy plate is maintained at a constant temperature T_{w} and the vapor is in a pure quiescent state at a uniform temperature T_{sat}, a liquid film of condensate will be formed on the plate surface when the wall temperature, T_{w}, is lower than the saturation temperature, T_{sat}. Due to the effects of capillary forces, a two-phase zone in which the liquid and the vapor coexist will then be formed between the liquid film and the vapor region.

Under steady-state conditions, the liquid film profile has a maximum height at the center of the plate and becomes progressively thinner toward the edge of the plate. The current analysis makes the assumptions that inertia and convection effects within the liquid film can be neglected and that the physical properties, the friction force and the ratio of the vapor density to the liquid density remain constant. Under these assumptions, the governing equations for the liquid film with boundary layer simplifications have the following forms:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

Momentum equation in x-direction:

$$0=-\frac{\partial P}{\partial x}-\frac{\mu_{\mathrm{e}}}{K}u \tag{3}$$

Momentum equation in y-direction:

$$0=-\frac{\partial P}{\partial y}-\rho g \tag{4}$$

According to Nusselt's classical analysis, the energy equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\int\limits_{w}^{w+\delta}\rho u\big(h_{\mathrm{fg}}+Cp(T_s-T)\big)\,\mathrm{d}y\right\}\mathrm{d}x+\rho h_{\mathrm{fg}}v_2|_{y_2=0}\,\mathrm{d}x = k_{\mathrm{e}}\frac{\partial T}{\partial y}\bigg|_{y=w}\mathrm{d}x\left[1+\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2\right]^{1/2} \tag{5}$$

where K is the intrinsic permeability of the porous medium; u and v are the Darcian velocity components in the x- and y-directions, respectively; P is the liquid pressure; δ is the liquid film thickness; μ_{e}, α_{e} and k_{e} are the effective dynamic viscosity, the effective thermal diffusivity and the effective thermal conductivity of a porous medium saturated with liquid, respectively; and $v_2|_{y_2=0}$ is the velocity of the liquid entering the two-phase zone as a result of the capillary suction effect, where $y_2=0$ corresponds to the interface of the liquid film and the two-phase zone.

The right-hand side of Eq. (5) represents the energy transferred from the liquid film to the wavy plate. Meanwhile, the first term on the left hand side of the equation describes the net energy flux across the liquid film(from x to $x+\mathrm{d}x$) and the second term expresses the net energy transferred into the two-phase zone. The boundary conditions of Eqs. (2)–(5) are specified as follows: $T=T_{\mathrm{w}}$ at $y=w(x)$ (the wavy plate surface) and $P=P_{\mathrm{sat}}$ at $y=\delta+w$ (the interface of the two-phase zone and the liquid film).

The static pressure, P, can be obtained by integrating Eq. (4) using the boundary condition $P=P_{\mathrm{sat}}$ at $y=\delta+w$, i.e.

$$P=P_{\mathrm{sat}}+\rho g(\delta+w-y) \tag{6}$$

Substituting Eq. (6) into Eq. (3) gives

$$u=-\frac{\rho g K}{\mu_{\mathrm{e}}}\frac{\mathrm{d}(\delta+w)}{\mathrm{d}x} \tag{7}$$

Since the thickness of the condensate film is small relative to the length of the plate, the temperature profile within the condensate film is assumed to be linear, i.e.

$$k_{\mathrm{e}}\frac{\partial T}{\partial y}\bigg|_{y=w}=k_{\mathrm{e}}\frac{\Delta T}{\delta} \tag{8}$$

In Eq. (5), the only remaining unknown is the capillary suction velocity ($v_2|_{y_2=0}$) of the liquid transferred into the two-phase zone.

At the interface of the two-phase zone and the liquid film, the mass continuity equation has the form

$$\frac{\partial u_2}{\partial x}\bigg|_{y_2=0}+\frac{\partial v_2}{\partial y_2}\bigg|_{y_2=0}=0 \tag{9}$$

At this interface, the no slip condition can be written as

$$u_2|_{y_2=0}=u|_{y=\delta+w} \tag{10}$$

or

$$\frac{\partial u_2}{\partial x}\bigg|_{y_2=0}=\frac{\partial u}{\partial x}\bigg|_{y=\delta+w} \tag{11}$$

Substituting Eq. (7) into Eq. (11), yields

$$\frac{\partial u_2}{\partial x}\bigg|_{y_2=0}=-\frac{\rho g K}{\mu_{\mathrm{e}}}\frac{\mathrm{d}^2(\delta+w)}{\mathrm{d}x^2} \tag{12}$$

Applying the principles of capillary pressure [24], the velocity in the two-phase zone can be expressed as

$$u_2=\frac{KK_{\mathrm{rl}}}{\mu_{\mathrm{e}}}\frac{\partial P_{\mathrm{c}}}{\partial x} \tag{13}$$

$$v_2=\frac{KK_{\mathrm{rl}}}{\mu_{\mathrm{e}}}\left(\frac{\partial P_{\mathrm{c}}}{\partial y_2}-\rho g\right) \tag{14}$$

where

$$K_{\mathrm{rl}}=s^3 \tag{15a}$$

$$P_{\mathrm{c}}=\frac{\sigma}{\sqrt{K/\varepsilon}}f(s) \tag{15b}$$

$$f(s)=1.417(1-s)-2.120(1-s)^2+(1-s)^3 \tag{15c}$$

s is the dimensionless saturation, and has a value of

$$s=1\quad \text{at } y_2=0 \tag{15d}$$

Substituting Eqs. (15a)–(15d) into Eq. (14), the capillary suction velocity in the two-phase zone, v_2, is given by

$$v_2=\frac{K}{\mu_{\mathrm{e}}}s^3\left(\frac{\sigma}{\sqrt{K/\varepsilon}}f'\frac{\partial s}{\partial y_2}-\rho g\right) \tag{16}$$

Substituting Eqs. (12) and (16) into Eq. (9) gives

$$-\frac{\rho g K}{\mu_e}\frac{d^2(\delta+w)}{dx^2}+\frac{K}{\mu_e}\frac{\sigma}{\sqrt{K/\varepsilon}}\left[s^3 f''\left(\frac{\partial s}{\partial y_2}\right)^2+3s^2 f'\left(\frac{\partial s}{\partial y_2}\right)^2 + f's^3\frac{\partial^2 s}{\partial y_2^2}-3\rho g s^2\left(\frac{\partial s}{\partial y_2}\right)\right]\bigg|_{y_2=0}=0 \tag{17}$$

According to Majumdar and Tien [26], the saturation profile near the interface of the two-phase zone and the liquid film is approximately linear, i.e.

$$\frac{\partial^2 s}{\partial y_2^2}=0 \quad \text{at } y_2=0 \tag{18}$$

Eq. (17) can therefore be re-written as

$$\frac{\partial s}{\partial y_2}\bigg|_{y_2=0} = \left(3\rho g s^2+\sqrt{\left(3\rho g s^2\right)^2+4\left(s^3 f''+3s^2 f'\right)\frac{\rho g\sigma}{\sqrt{K/\varepsilon}}\frac{d^2(\delta+w)}{dx^2}}\right) \times\left(2\frac{\sigma}{\sqrt{K/\varepsilon}}\left(s^3 f''+3s^2 f'\right)\right)^{-1}\Bigg|_{y_2=0} \tag{19}$$

Substituting Eq. (19) into Eq. (16), the velocity of the liquid entering the two-phase zone from the liquid film as a result of capillary effects is expressed as

$$v_2|_{y_2=0}=\frac{K}{\mu_e}s^3 \times\left(f'\left(3\rho g s^2+\sqrt{\left(3\rho g s^2\right)^2+4\left(s^3 f''+3s^2 f'\right)\frac{\rho g\sigma}{\sqrt{K/\varepsilon}}\frac{d^2(\delta+w)}{dx^2}}\right) \times\left(2\left(s^3 f''+3s^2 f'\right)\right)^{-1}-\rho g\right)\Bigg|_{y_2=0} \tag{20}$$

Substituting Eqs. (7), (8) and (20) into Eq. (5) yields

$$\delta\frac{d}{dx}\left(\delta\frac{d(\delta+w)}{dx}\right)-\delta\left(\frac{h_{fg}}{h_{fg}+\frac{1}{2}Cp\Delta T}\right) \times\left(\left(3f's^5+s^3 f'\sqrt{9s^4+4\left(s^3 f''+3s^2 f'\right)\frac{\sigma}{\sqrt{K/\varepsilon}}\frac{1}{\rho g}\frac{d^2(\delta+w)}{dx^2}}\right) \times\left(2\left(s^3 f''+3s^2 f'\right)\right)^{-1}-s^3\right)\Bigg|_{y_2=0} = \frac{-k_e\Delta T\mu_e}{\rho^2 g K\left(h_{fg}+\frac{1}{2}Cp\Delta T\right)}\left[1+\left(\frac{dw}{dx}\right)^2\right]^{1/2} \tag{21}$$

For analytical convenience, the following dimensionless parameters are introduced:

$$Ja=\frac{Cp\Delta T}{h_{fg}+\frac{1}{2}Cp\Delta T},\quad Ra_e=\frac{\rho^2 g\,Pr_e L^3}{\mu_e^2},\quad Pr_e=\frac{\mu_e Cp}{k_e}$$
$$Da=\frac{K}{L^2},\quad Bo_c=\frac{\sigma\sqrt{\varepsilon}}{\rho g K}=\frac{1}{Bo},\quad F=\frac{a}{L} \tag{22}$$

In Eq. (22), Bo_c is a dimensionless capillary parameter defined as the ratio of the surface tension force to the gravitational force. If Bo_c is large, the effects of the surface tension force are relatively greater than those of gravity, and hence the two-phase zone contributes significantly to the overall liquid flow. Written in terms of the non-dimensional variables given in Eq. (22), Eq. (21) becomes

$$\delta\frac{d}{dx}\left(\delta\frac{d(\delta+w)}{dx}\right)-\delta\left(1-\frac{1}{2}Ja\right) \times\left(\left(3f's^5+f's^3\sqrt{9s^4+4\left(s^3 f''+3s^2 f'\right)Bo_c\sqrt{K}\frac{d^2(\delta+w)}{dx}}\right) \times\left(2\left(s^3 f''+3s^2 f'\right)\right)^{-1}\right)\Bigg|_{y_2=0}+\left(1-\frac{1}{2}Ja\right)\delta\times s^3\Big|_{y_2=0} =\frac{-Ja}{Ra_e}\frac{L}{Da}\left[1+\left(\frac{dw}{dx}\right)^2\right]^{1/2} \tag{23}$$

with the following boundary conditions:

$$\frac{d(\delta+w)}{dx}=0 \quad \text{at } x=0 \tag{24a}$$

$$\delta=\delta_{\min} \quad \text{at } x=L \tag{24b}$$

However, even with these boundary conditions, Eq. (23) still cannot be solved since $\delta_{\min}$ is unknown.

In practice, the film thickness at the plate edge cannot be exactly zero. Its value can be calculated by applying the minimum mechanical energy principle presented by Bakhmeteff [15] for open channel hydraulics. This principle states that a fluid flowing across a hydrostatic pressure gradient and over the edge of a plate will adjust itself such that the mechanical energy within the fluid is minimized with respect to the boundary layer at the plate edge. The minimum(i.e. critical)thickness can therefore be calculated by setting the derivative of the mechanical energy with respect to δ equal to zero for steady-state flow rate conditions, i.e.

$$\left[\frac{\partial}{\partial\delta}\int_w^{w+\delta}\left(\frac{u^2}{2}+gy+\frac{P}{\rho}\right)\rho u\,dy\right]\bigg|_{\dot m_c=0} \tag{25}$$

where $\dot m_c$ is the critical value of the mass flow over the plate edge.

Substituting Eqs. (6) and (7) into Eq. (25) yields the new boundary condition

$$\frac{d(\delta+w)}{dx}\bigg|_{x=L}=-\left(\frac{\mu_e^2}{\rho^2 g K^2}\delta_{\min}\right)^{1/2} \tag{26}$$

By introducing the normalized variables $x^*=x/L$, $\delta^*=\delta/L$ and $w^*=w/L$, Eq. (23) can be re-written as

$$\delta^*\frac{d}{dx^*}\left(\delta^*\frac{d(\delta^*+w^*)}{dx^*}\right)-\delta^*\left(1-\frac{1}{2}Ja\right) \times\left(\left(3f's^5+f's^3\sqrt{9s^4+4\left(s^3 f''+3s^2 f'\right)Bo_c Da^{1/2}\frac{d^2(\delta^*+w^*)}{d^2x^*}}\right) \times\left(2\left(s^3 f''+3s^2 f'\right)\right)^{-1}\right)\Bigg|_{y_2=0}+\left(1-\frac{1}{2}Ja\right)\delta^*\times s^3\Big|_{y_2=0} =\frac{-Ja}{Ra_e Da}\left[1+\left(\frac{dw^*}{dx^*}\right)^2\right]^{1/2} \tag{27}$$

Eq. (24a) becomes

$$\frac{d\delta^*}{dx^*}=0 \quad \text{at } x^*=0 \tag{28a}$$

and Eq. (26) becomes

$$\frac{d(\delta^*+w^*)}{dx^*}\bigg|_{x^*=1}=-\left(\frac{Pr_e}{Ra_e Da^2}\delta^*_{\min}\right)^{1/2} \tag{28b}$$

The values of s, f' and f'' at $y_2 = 0$ are calculated from Eqs. (15c) and (15d) to be 1, -1.417 and -4.24, respectively. Finally, with $w^* = w/L = F\cos(n\pi x^*)$, applying Eq. (27) and the related boundary condition equations, i.e. Eqs. (28a) and (28b), the dimensionless film thickness, δ^*, can be expressed in terms of Ja, Ra_e, Pr_e, Bo_c, Da, F and n.

In this study, the dimensionless liquid film thickness is obtained by solving Eq. (27) numerically using the Runge–Kutta shooting method. In the solution procedure, an initial estimate is made of the liquid film thickness at the center of the plate ($\delta^*|_{x^*=0}$) and this estimate is then substituted into Eq. (27). Applying Eq. (28a), the variation of δ^* along the x^* direction is then calculated. Once the value of δ^* at each grid point has been computed, the resulting thickness gradient at the plate edge is checked using Eq. (28b) and a modified value of $\delta^*|_{x^*=0}$ is obtained. This value is then substituted back into Eq. (27) and the liquid film thickness across the surface of the plate is recalculated. This process is repeated iteratively until the condition of Eq. (28b) satisfies the convergence criterion, i.e.

$$\left|\frac{\mathrm{d}(\delta^* + w^*)/\mathrm{d}x^*|_{x^*=1} + (Pr_e/(Ra_e Da^2)\delta^*_{\min})^{1/2}}{\mathrm{d}(\delta^* + w^*)/\mathrm{d}x^*|_{x^*=1}}\right| \leqslant 10^{-6}.$$

The local Nusselt number is calculated as

$$Nu_x = \frac{h(x)L}{k_e} \tag{29}$$

where

$$h(x) = \frac{k_e[1 + (\mathrm{d}w/\mathrm{d}x)^2]^{1/2}}{\delta} \tag{30}$$

The mean Nusselt number is given by

$$\overline{Nu} = \frac{\bar{h}L}{k_e} \tag{31}$$

where

$$\bar{h} = \frac{1}{L}\int_0^L h(x)\,\mathrm{d}x = \frac{1}{L}\int_0^L \frac{k_e[1 + (\mathrm{d}w/\mathrm{d}x)^2]^{1/2}}{\delta}\,\mathrm{d}x$$
$$= \frac{1}{L}\int_0^1 \frac{k_e[1 + (\mathrm{d}w^*/\mathrm{d}x^*)^2]^{1/2}}{\delta^*}\,\mathrm{d}x^* \tag{32}$$

3. Results and discussion

Wang et al. [22] used a novel transformation method to investigate the problem of film condensation on a horizontal wavy plate in a porous medium under the assumption of negligible capillary force effects. In the analysis developed in the present study, the effects of capillary forces on the film condensation can be removed from the model by specifying the capillary suction velocity $v_2|_{y_2=0}$ as zero. Under these conditions, the energy balance equation (Eq. (27)) can be rewritten as

$$\delta^* \frac{\mathrm{d}}{\mathrm{d}x^*}\left(\delta^* \frac{\mathrm{d}(\delta^* + w^*)}{\mathrm{d}x^*}\right) = \frac{-Ja}{Ra_e Da}\left[1 + \left(\frac{\mathrm{d}w^*}{\mathrm{d}x^*}\right)^2\right]^{1/2} \tag{33}$$

Note that the corresponding boundary conditions are the same as those given in Eqs. (28a) and (28b).

Table 1
Comparison of solutions presented by Wang et al. [22] and those computed by Eq. (33) for the specific case of negligible capillary effects

Parameters	$\overline{Nu}$ (Wang et al.)	$\overline{Nu}$ (present study)
$Ja = 0.01$, $Ra_e = 10^5$, $Da = 0.01$, $Pr_e = 0.1$	36.0	36.8
$Ja = 0.01$, $Ra_e = 10^5$, $Da = 0.01$, $Pr_e = 7$	55.7	59.7
$Ja = 0.01$, $Ra_e = 10^5$, $Da = 0.01$, $Pr_e = 100$	61.9	61.9
$Ja = 0.01$, $Ra_e = 10^5$, $Da = 0.05$, $Pr_e = 7$	75.8	78.9

Table 2
Physical parameters used in current analysis

Dimensional parameters	
Permeability $K = 6.4 \times 10^{-12}$ m^2	Porosity $\varepsilon = 0.38$
Liquid density $\rho = 957.9$ kg/m^3	Specific heat at constant pressure $Cp = 4217$ J/kg °C
Liquid viscosity $\mu_e = 2.79 \times 10^{-4}$ kg/m s	Heat of vaporization $h_{fg} = 2257$ kJ/kg
Surface tension $\sigma = 5.94 \times 10^{-2}$ N/m	Half-width of plate $L = 0.1$ m
Dimensionless parameters	
$Ja = \frac{Cp\Delta T}{h_{fg} + \frac{1}{2}Cp\Delta T} = 0.05$	$Ra_e = \frac{\rho^2 g Pr_e L^3}{\mu_e^2} = 2 \times 10^{11}$
$Pr_e = \frac{\mu_e Cp}{k_e} = 1.76$	$Bo_c = \frac{\sigma\sqrt{\varepsilon}}{\rho g K} = 6.1 \times 10^5$
$Da = \frac{K}{L^2} = 6.4 \times 10^{-10}$	$F = \frac{a}{L} = 0.01$

Table 1 compares the results presented by Wang et al. in [22] for the mean Nusselt number with those computed using Eq. (33). From inspection, the maximum difference between the two sets of results is found to be less than 6.5%.

The following analyzes consider the same sand–water–steam system as that used by Udell in [24]. This system is chosen specifically here since water–steam is the most commonly employed working liquid in practical engineering problems. Table 2 summarizes the dimensional and dimensionless parameter values used in the present analysis.

Fig. 2 illustrates the distribution of the dimensionless liquid film profiles across the plate surface for wavy surfaces with $n = 0, 1, 2, 3$ and 4, respectively. Note that the surface profiles are also presented for reference purposes. The dimensionless parameters are assigned the typical values shown in Table 2, i.e. $Da = 6.4 \times 10^{-10}$, $Ra_e = 2 \times 10^{11}$, $Ja = 0.05$, $Pr_e = 1.76$, $Bo_c = 6.1 \times 10^5$, and $F = 0.01$. It can be seen that the dimensionless liquid film profiles on the surfaces with $n = 2$ and $n = 4$, respectively, are higher than those on either the smooth plate, i.e. $n = 0$, or on the wavy plates with $n = 1$ or $n = 3$, respectively. This result arises since the surface profiles at the edges of the plates with $n = 2$ and $n = 4$ are higher than those at the edges of the smooth plate or the wavy plates with $n = 1$ or $n = 3$. This implies that a greater volume of liquid condensate accumulates on a wavy plate with a higher plate edge ($n = 2, 4$). In other words, the liquid condensate flows more easily across the wavy surface when the wavy plate has a lower edge height ($n = 1, 3$).

Figs. 3 and 4 plot the variation of the local dimensionless condensate thickness, δ^*, with x^* for wavy plates with even and odd wave numbers, respectively. The two figures also plot

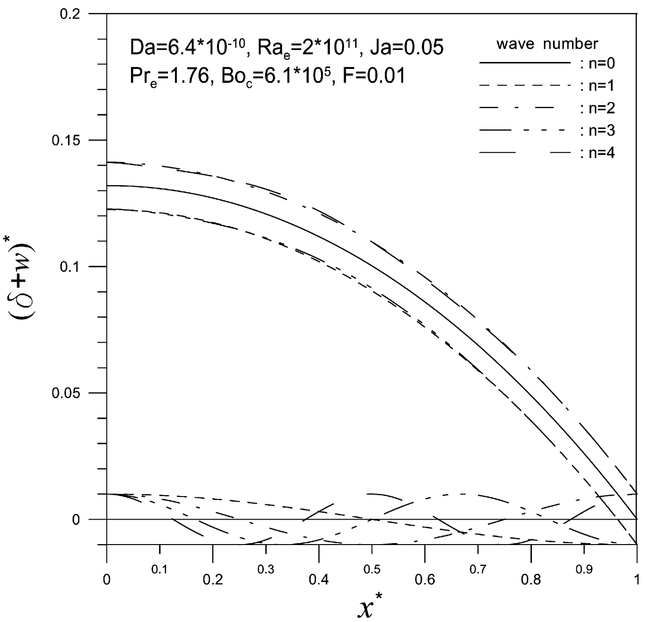

Fig. 2. Distribution of dimensionless liquid film profiles across wavy surfaces with wave numbers of $n = 0, 1, 2, 3$ and 4. (Note: corresponding surface profiles are also shown for reference purposes.)

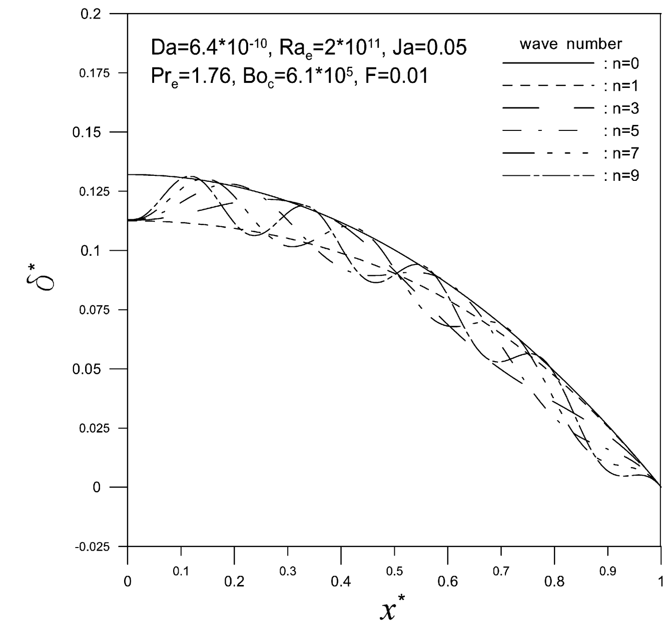

Fig. 4. Distribution of dimensionless liquid film thickness, δ^*, across surface profiles with odd wave numbers.

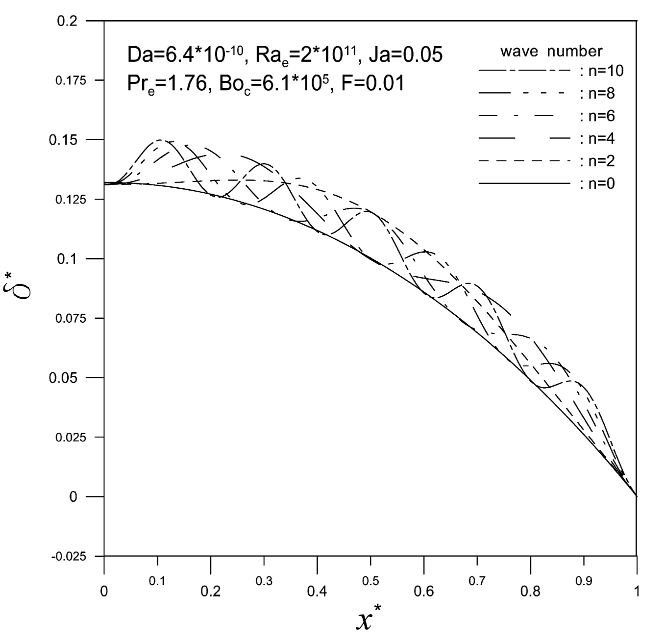

Fig. 3. Distribution of dimensionless liquid film thickness, δ^*, across surface profiles with even wave numbers.

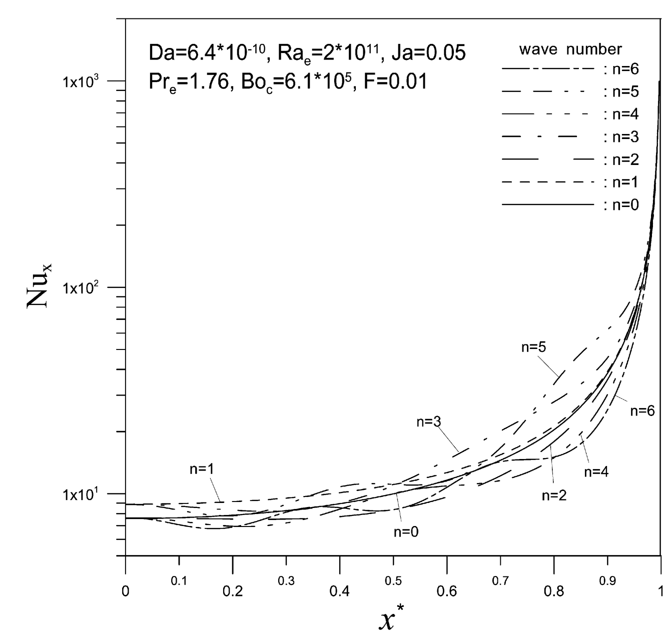

Fig. 5. Distribution of local Nusselt number, Nu_x, across surface profile.

the case of a smooth plate ($n = 0$) for comparison purposes. In both figures, it can be seen that the wave number of the dimensionless liquid film thickness matches that of the wavy plate. In addition, Fig. 3 shows that the dimensionless liquid film thickness on a wavy plate with an even wave number is greater than that on a smooth plate. Conversely, Fig. 4 shows that the dimensionless liquid film thickness on a smooth plate is greater than that on a wavy plate with an odd wave number. These results are explained by the fact that a lower surface height at the plate edge (which occurs when the wave number is odd) induces a stronger gravity force effect and hence reduces the dimensionless liquid film thickness. In other words, a plate with a filleted edge (i.e. with a reduced edge height) can be used to assist the condensate liquid to flow over the plate edge and therefore to reduce the liquid film thickness.

Substituting the local dimensionless film thickness into Eqs. (30) and (32), the local and mean Nusselt numbers can be derived from Eqs. (29) and (31), respectively. Fig. 5 shows the distribution of the local Nusselt number, Nu_x, across the surface of the wavy plate as a function of the wave number, n, for $Da = 6.4 \times 10^{-10}$, $Ra_e = 2 \times 10^{11}$, $Ja = 0.05$, $Pr_e = 1.76$, $Bo_c = 6.1 \times 10^5$, and $F = 0.01$. As shown in Eq. (30), the value of Nu_x increases as the dimensionless liquid film thickness de-

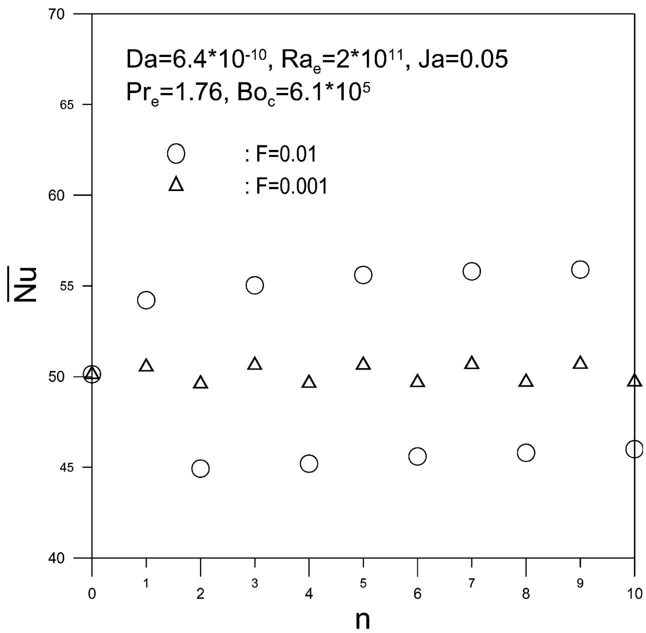

Fig. 6. Variation of mean Nusselt number, $\overline{Nu}$, with wave number, n, for $Ja = 0.05$, $Ra_e = 2 \times 10^{11}$, $Da = 6.4 \times 10^{-10}$, $Pr_e = 1.76$, and $Bo_c = 6.1 \times 10^5$.

creases. As expected, the wavy plates with an odd wave number result in an enhanced heat transfer performance.

Fig. 6 shows the variation of the mean Nusselt number, $\overline{Nu}$, with the wave number, n, for dimensionless wave amplitudes of $F = 0.001$ and 0.01, respectively, for $Da = 6.4 \times 10^{-10}$, $Ra_e = 2 \times 10^{11}$, $Ja = 0.05$, $Pr_e = 1.76$ and $Bo_c = 6.1 \times 10^5$. It is observed that the value of $\overline{Nu}$ increases slightly with an increasing wave number. It is also seen that when the dimensionless wave amplitude is small ($F = 0.001$), the value of $\overline{Nu}$ is close to that of a smooth plate. However, the edge shape of the wavy plate has a significant effect on the mean Nusselt number. Specifically, as mentioned above, a plate with a filleted edge assists the condensate liquid to flow over the plate edge and therefore enhances the rate of condensation heat transfer.

In practical applications of water vapor condensing on a horizontal wavy plate in a porous medium, engineers must decide whether the effect of capillary forces on the heat transfer performance should be taken into consideration. Figs. 7 and 8 show the variation of the Nusselt number, $\overline{Nu}$, with the capillary parameter, Bo_c, as a function of Ja for wavy plates with wave numbers of $n = 0$ and 7, respectively, and constant parameter values of $Da = 6.4 \times 10^{-10}$, $Ra_e = 2 \times 10^{11}$ and $Pr_e = 1.76$. In both figures, it is seen that $\overline{Nu}$ increases only slowly with increasing Bo_c when $Bo_c < 10^3$. In other words, the capillary force has only a negligible effect on the heat transfer performance and can therefore be neglected. However, when $Bo_c > 10^3$, $\overline{Nu}$ increases significantly with increasing Bo_c. In other words, $Bo_c = 10^3$ represents a threshold value above which the effects of capillary forces should be taken into account. The physical reason for the enhanced heat transfer at higher values of Bo_c is that the stronger capillary forces result in more liquid being sucked from the condensate layer into the two-phase zone. Hence, the thickness of the liquid film is reduced and the temperature gradient is increased, both of which enhance the heat transfer rate. For the current problem of water vapor condensing on a horizontal wavy plate in a porous medium, the value of Bo_c $(= \sigma\sqrt{\varepsilon}/(\rho g K))$ is 6.1×10^5, which is clearly far higher than 10^3. Therefore, the effects of capillary forces must be taken into account. Figs. 7 and 8 also reveal that $\overline{Nu}$ decreases as Ja increases. This result is consistent with the findings presented in [13,14,21], and arises because a higher value of Ja implies a higher value of ΔT, which in turn increases the average condensate film thickness and therefore reduces the heat transfer performance.

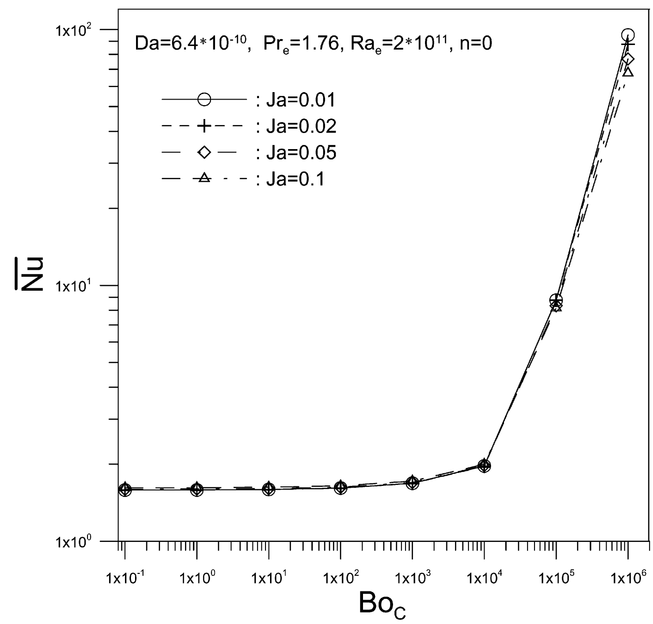

Fig. 7. Variation of mean Nusselt number, $\overline{Nu}$, with Ja for surface profile with $n = 0$.

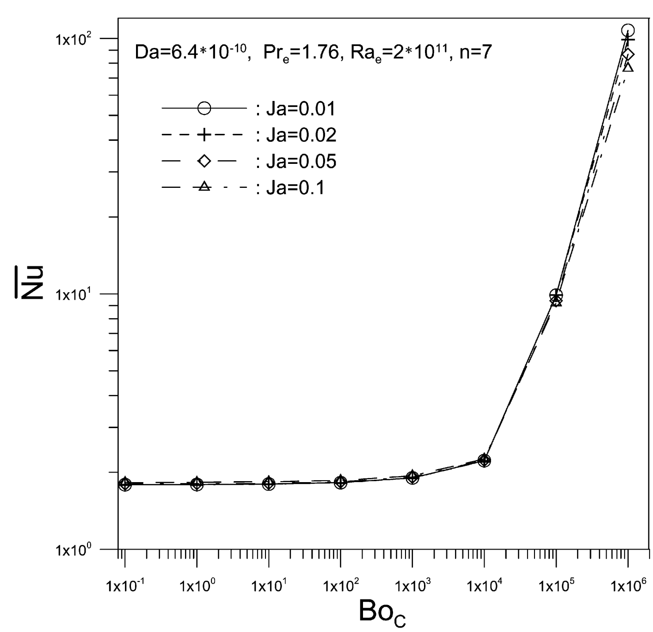

Fig. 8. Variation of mean Nusselt number, $\overline{Nu}$, with Ja for surface profile with $n = 7$.

4. Conclusion

This study has considered the problem of laminar film condensation on a horizontal wavy plate embedded in a porous medium. The presence of capillary effects results in the formation of a two-phase zone immediately above the liquid film. Furthermore, the capillary effects reduce the thickness of the liquid film and hence increase the heat transfer coefficient. The results have indicated that a value of $Bo_c = 10^3$ represents a threshold value for the capillary parameter above which capillary force effects must be taken into consideration. It has also been shown that both the wave number and the wave amplitude of the wavy plate have a significant effect on the mean Nusselt number, $\overline{Nu}$. Finally, wavy plates with an odd wave number result in an enhanced heat transfer performance. In other words, the rate of condensation heat transfer can be enhanced by introducing a fillet at the edge of the wavy plate.

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